Abstract

$k^+\text{-buffer}$ facilitates novel approaches to multi-fragment rendering and visualization for developing interactive applications on the GPU. Various alternatives have been proposed to alleviate its memory hazards and to avoid completely or partially the necessity of geometry pre-sorting. However, that came with the burden of excessive memory allocation and depth precision artifacts. We introduce $k^+\text{-buffer}$, a fast and accurate framework that simulates the $k$-buffer behavior by exploiting fragment culling and pixel synchronization. Two GPU-accelerated data structures have been developed: (i) the max-array and (ii) the max-heap. These memory-bounded data structures accurately maintain the $k$-foremost fragments per pixel in a single geometry pass. The choice of the data structure depends on the size $k$ (application-dependent). Without any software-redesign, the proposed scheme can be adapted to perform as a Z-buffer or an A-buffer capturing a single or all generated fragments, respectively. A memory-friendly strategy is also proposed, extending the proposed pipeline to dynamically lessen the potential wasteful memory allocation. Finally, an extensive experimental evaluation is provided demonstrating the advantages of $k^+\text{-buffer}$ over all prior $k$-buffer variants in terms of memory usage, performance cost and image quality.

CR Categories: I.3.6 [Computer Graphics]: Methodology and Techniques—Graphics data structures and data types

Keywords: GPU, $k$-buffer, a-buffer, max-heap, multi-fragment effects, real-time, pixel synchronization

1 Introduction

Determining visibility when rasterizing objects in arbitrary order is a challenging task in terms of time and space for a host of algorithms that simulate complex rendering effects in real-time. Many image-based techniques produce visually realistic results at interactive speeds in games (order-independent transparency [Maule et al. 2011], shadows [Yang et al. 2010], hair rendering [Yu et al. 2012]) and other graphics applications (volume rendering [Callahan et al. 2005], collision detection [Jang and Han 2008], constructive solid geometry [Rossignac et al. 2013]) enabled by a family of GPU-accelerated methods that capture the surface-intersections (fragments) when ray casting from the viewer position through each screen pixel. Figure 2 shows an illustrative example of the fragment generation process through per-pixel ray-surface intersection tests. Buffer-based techniques ensure the correct visibility order on the level of fragments, avoiding drawbacks (e.g. geometry interpenetration, primitive splitting, dynamic scenes) that occur in object/primitive sorting techniques [Govindaraju et al. 2005; Sintorn and Assarsson 2008].

Extending Z-buffer, where the closest-to-viewer fragment is stored, A-buffer [Carpenter 1984] was the first method to capture all fragments per pixel in a single geometry pass. Fragments are stored into variable-length linked lists per pixel during geometry rendering, followed by a post-sorting process that correctly reorders fragments by their depth. Several variants and optimizations [Maule et al. 2011] have been proposed recently to simulate the behavior of the A-buffer architecture with reduced memory requirements generating more or less approximate image outputs.

$k$-buffer [Bavoil et al. 2007] employed per-pixel fixed-size vectors maintaining a $k$-nearly sorted sequence of fragments in bounded GPU memory. When the number of generated fragments exceeds $k$, the rasterization order of the generated fragments affects the final result due to memory read-modify-write (RMW) hazards. Heuristics (primitive pre-sorting and triangle batches) may alleviate most of the image quality flaws with the burden of higher computational cost. A number of GPU-accelerated $k$-buffer variants have been recently introduced that avoid RMW hazards and geometry ordering prior to rasterization by either initially building an A-buffer [Yu et al. 2012] or exploring a two-geometry-pass scheme based on atomic memory operations [Liu et al. 2010; Maule et al. 2013]. However, a significant increase of unbounded memory is required when A-buffer is used as temporary storage. On the other hand, atomic operations are restricted to only work on 32-bit unsigned integer format resulting in (i) visual artifacts from lost precision when converting depth values and (ii) performance cost of an additional geometry pass.

We introduce $k^+\text{-buffer}$ ($K^+\text{B}$), a single-geometry-pass $k$-buffer framework that overcomes the aforementioned performance bottlenecks, memory footprints and rasterization artifacts. Inspired by [Crassin 2010b], a semaphore-based spin-lock mechanism ensures atomicity of the per-pixel fragment operations at the shared memory. Implementation details are also provided to easily switch to the hardware-implemented pixel syncing solution available on the
Haswell architecture [Salvi 2013]. To alleviate contention (busying-
waits) of distant fragments, we concurrently perform calling
checks that efficiently discard fragments that are further from all
currently maintained fragments. Two array-based data structures are
built on the GPU to accurately store the closest per-pixel fragments:
(i) max-array, an array where the maximum element is always stored
at the first entry and (ii) max-heap, a complete binary tree in which
the value of each internal node is greater than or equal to the values
of the children of that node. Despite its linear complexity, the former
performs faster than the latter when the problem size is sufficiently
small. For example, order-independent transparency presents
high approximation images even with a small core of captured
layers ($k \leq 16$). Conversely, plausible photorealistic appearance
of hair requires the contribution of a larger set of hair strands ($k > 16$).

To avoid the wasteful pre-allocated storage requirements of
pixels that contain less than $k$ fragments ($k$-fragmentless pixels, see
Figure 2), we have extended our framework by the S-buffer
pipeline [Vasilakis and Fudos 2012]. An additional geometry pass is
performed for counting fragments per pixel, enabling us to allocate
the exact amount of memory that we actually need. Memory is
linearly organized into variable contiguous regions for each pixel,
making it feasible to implement both proposed data structures. To
our knowledge, this is the first $k$-buffer implementation with dy-
namic and precise allocation of the required storage space. Without
any adaptation, our framework can also support the functionalities
of the Z-buffer and A-buffer by setting $k$ to either one or large
enough to handle all possible per-pixel fragment cardinalities.

The structure of this paper is as follows: Section 2 offers a brief
overview of prior art. Section 3 introduces the algorithmic details of
our $k^1$-buffer framework. Section 4 provides extensive comparative
results for several of multi-fragment rendering alternatives. Finally,
Section 5 offers conclusions and identifies future research directions.

Figure 2: Illustrating the construction process of a row of a
4-buffer (highlighted with blue at the top-right thumbnail), when ray
casting the dragon model. A significant amount of memory space
is wasted at pixels that consist of less than 4 fragments due to the
pre-allocation of the same buffer length per pixel.

2 Related Work

There is an abundance of recent research on treating the problem
of storing multiple fragments from different perspectives. A-
buffer [Carpenter 1984] exploited an unbounded buffer for building
dynamically created per pixel lists of all fragments in submission
order using a single rendering pass. Then, stored fragments are post-
sorted according to their depth values. An actual GPU-accelerated
implementation of A-buffer based on atomic memory operations
was recently introduced in [Yang et al. 2010] (LL). However, its
performance degrades rapidly due to the heavy contention and the
random memory accesses when constructing and assembling the
entire fragment list, respectively. A memory-friendly variation
of this algorithm was described in [Crassin 2010b], where pagged
per-pixel linked-lists (LL-Paged) improve caching and data bus
occupancy. On the other hand, [Vasilakis and Fudos 2013] improved
performance by concurrently storing fragments into more than one
per-pixel linked lists. They have proposed to uniformly split the
depth range of the rasterized scene (see also [Liu et al. 2009b; Sintosh
and Assarsson 2009]) and assign one linked list in each subdivision.

FreePipe [Liu et al. 2010], a complex CUDA-based rasterization
pipeline, maintains multiple fragments using constant-size per pixel
vectors. To ensure complete fragment extraction for all pixels, the buffer length must be set accurately. FreePipe has been realized
using modern OpenGL APIs [Crassin 2010a], thus avoiding switch-
ing from the traditional graphics pipeline to a software rasterizer.
Despite its high computation speed, FreePipe suffers from large and
potentially unnecessary memory consumption. Last but not least,
S-buffer (SB) [Vasilakis and Fudos 2012], a two-geometry-passes
A-buffer implementation on the GPU, has overcome the limitations
of both linked-lists and fixed-array techniques by taking advantage of
the fragment distribution and the sparsity of the pixel-space.

Regardless of the data structure, the aforementioned approaches suffer from (i) memory overflows resulted from the unbounded
buffer needed to store all generated fragments, and (ii) performance bottle-necks arise when the number of per-pixel fragments to be post-
sorted increases significantly. To this end, k-buffer (KB) [Callahan
et al. 2005; Bavoil et al. 2007] reduced memory requirements
by capturing the k-closest to the viewer fragments in a single geometry
rasterization. However, it is susceptible to disturbing flickering art-
ifacts caused from RMW hazards during fragment insertion updates.
[Liu et al. 2009a] extended this work to a multi-pass approach (KB-
Multi) achieving robust rendering behavior with the trade-off of low
frame rates. Moreover, [Bavoil and Myers 2008] eliminated most of
the memory conflicts by performing stencil routing on a multi-
sample anti-aliasing buffer (KB-SR). Finally, [Zhang 2013] explored
a memory-hazard-aware solution (KB-MHA) based on a depth-
error correction coding scheme. In practice however, they
cannot guarantee correct results in all cases. The image quality of
this class of methods is highly dependent on a coarse CPU-based
pre-sorting in primitive-space which eliminates the arrival of out-of-
order fragments. Multiple rendering iterations, are further required
to provide an A-buffer output, due to the limited number of multiple
render targets on the GPU. This results in significant performance
downgrade. Conversely, [Wang and Xie 2013] proposes partitioning
the input scene into components with a few layers and then rendering
them individually in order to fit into the limited KB-SR buffer size.
However, this comes with the burden of non-supporting animated
scenes and having limited order-dependent applicability.

Multi depth test scheme (KB-MDT), developed in both CUDA [Liu
et al. 2010] and OpenGL [Bavoil et al. 2013] APIs, guarantees correct
depth order results by capturing and sorting fragments on the fly via
32-bit atomic integer comparisons. Since 64-bit atomic operations
are not supported from available APIs to update the depth and color
buffers simultaneously, a costly additional geometry pass is therefore
suggested. Furthermore, noisy images may be generated due to the
precision lost when converting floating depth values of close frag-
ments. Similar to our method, [Salvi 2013] extended the original k-
buffer to avoid fragment racing by employing hardware-aware pixel
synchronization (KB-PS). However, this method is currently com-
patible only with graphics cards based on the Haswell architecture.

Finally, [Yu et al. 2012] proposed two linked-list-aware solutions to
accurately compute the k-foremost fragments. The idea of the first
one is to capture all fragments by initially constructing A-buffer by
the method of [Yang et al. 2010], followed by a step that selects and
sorts the k-nearest fragments (KB-AP). On the other hand, the
second approach directly computes depth-ordered per-pixel linked
lists avoiding the unnecessary A-buffer construction (KB-LL).
Despite the fact that the second approach requires less storage,
fragments are sparsely stored in graphics memory causing the
additional allocation of contiguous blocks of memory.
Table 1 presents a comparative overview of all k-buffer alternatives with respect to memory requirements, rendering complexity, fragment extraction accuracy, and sorting necessity.

3 Framework Overview

We propose an efficient k-buffer implementation on the GPU which is free from: (i) geometry sorting prior rasterization, (ii) unbounded memory necessity, (iii) RMW memory-hazards, and (iv) depth precision conversion artifacts. Contrary to most of the aforementioned k-buffer alternatives which store and sort the generated fragments on the fly, we follow a faster strategy similar to the one used by the A-buffer construction: The k-nearest fragments are captured in an unsorted sequence, followed by a post-sorting step that reorders them by their depth. We explore a GPU-accelerated spin-lock strategy via pixel semaphores to ensure real-time synchronized construction of the unsorted k-front fragments (Section 3.1). Two bounded array-based data structures for fragment data are introduced which enable a low cost culling test that concurrently discards outlier fragments (Section 3.2). A pipeline extension is further introduced to dynamically and precisely handle graphics memory allocation (Section 3.3). k^-buffer can also be considered as an unified framework that successfully integrates the functionalities of Z-buffer, k-buffer, and A-buffer (Section 3.4). The overall framework is described by offering shader-like pseudocode and the fragment processing pipeline. Finally, we highlight features and tradeoffs of our framework, pointing out implementation details and lightweight modifications that can be used to guide the decision of which pipeline alternative to employ in a given setting.

3.1 Spin-lock Strategy

Per-pixel binary semaphores are utilized as a synchronization mechanism to ensure fragment exclusive use of the critical storage section. Taking into account the possibility of simultaneous access to the lock, which could cause race conditions, an implementation of an atomic text-and-set operation is explored. The culling process obtains the lock if the old value was 0. It spins writing 1 to the variable until this occurs. One way to implement spin-lock strategy employing text-and-set into a pixel shader is shown in the Algorithm 1.

Algorithm 1 MutualExclusion (Texture t, Pixel p)

1: while true do ▷ spin until lock is free
2: if imageAtomicExchange(t, p, 1) == 0 then
3: (critical section) ▷ exclusive use
4: imageStore(t, p, 0); ▷ release lock when finished
5: discard; ▷ exit shader
6: end if
7: end while

A 32-bit unsigned integer texture with internal pixel format R_32UI is allocated to represent the per-pixel semaphores. At first, a full-screen rendering (clear pass) is executed to initialize texture with zeros. Our method is enhanced by the OpenGL’s imageAtomicExchange(texture lock, tvec2 P, uint V) function which atomically replaces the value V of the atomic object with the argument into texel at coordinate P and returns its original value. Note that there is no need for an atomic operation to perform the lock release (since the running fragment has exited from the critical section) as opposed to the implementation of [Crassin 2010b] where an additional costly atomic exchange is used.

Pixel Synchronization (PS) is a graphics extension that Intel has implemented for 4th Generation Intel Core processors with Iris and Iris Pro graphics based on Haswell architecture. PS provides a performance-wise inexpensive mechanism which avoids fragment conflicts in the critical section and ensures that RMW memory operations are performed in submission order [Salvi 2013]. Our framework can be enhanced by the use of PS without remodeling the proposed pipeline. Implementation-wise, a simple call of begin-FragmentShaderOrderingINTEL() function is necessary to provide fragment serializability. Thus, the per-pixel semaphore-based spin-lock strategy can be omitted (specifically, lines 18-19 and 25-28 in Algorithm 3). Avoiding the usage of per-pixel semaphores also results in reduced memory demand. While DirectX11+ and OpenGL extensions are available for Intel graphics cards, we expect that these will be supported in the near future by all manufacturers.

3.2 Fragment Capturing

A geometry rendering (store pass) is initially carried out to capture the closest fragment data per-pixel in a 64-bit floating point 3D array buffer with internal format of RG_32F, (R for color and G for depth) and k length. Figure 2 illustrates a k^-buffer which can hold up to 400 fragments (screen size: 10 × 10, k = 4).

To alleviate the spinning of n generated fragments that do not belong to the closest k, a fast culling mechanism is performed. The idea is to efficiently discard each incoming fragment fi, ∀i ∈ {0, . . . , n − 1} that has equal or larger depth value (f_i, z) from all currently maintained fragments, before trying to acquire the semaphore. Note that determines the submission order. Let KB[i] = [kB[i]z | kB[i]z = 0, k], k ≥ KB[i]z) and swap it with the newly added fragment (except if the tree root is at index 0(k)). Then, the heap property is restored by traversing the entire pixel row for every incoming fragment, we have developed two array-based data structures on the GPU that both store the maximum element at the first array position: (i) max-array (K^-B-Array) and (ii) max-heap (K^-B-Heap). Thus, this operation is performed in constant time. Figure 3 illustrates how two incoming fragments are successfully discarded using this formula when the buffer is completely full.

Max-array can be considered as an array where the fragment with the largest depth value is always stored at the first location and the rest are randomly positioned. When an incoming fragment obtains a semaphore, it stores its information in the first empty entry (O(1)). In this case, a per-pixel counter (32-bit unsigned integer texture with internal pixel format R_32UI) is utilized as index and incremented after a successful insertion. Per-pixel counters are initialized to zero during the clear full-screen rendering pass. If the array is full (counter == k), it takes the place of the fragment with the larger depth value. Note that since culling mechanism resides outside critical section, an additional checking is mandatory to guarantee correct results. To keep max-array consistent after an insertion on a completely filled array, we find the fragment with the largest depth value (O(k)) and swap it with the newly added fragment (except the latter is the largest one). This process is implemented without the use of any costly atomic memory operations since fragment atomicity is guaranteed.

However when the problem size increases rapidly (k > 16), fragment data information is maintained in a max-heap data structure. Max-heap is a complete binary tree (shape property) in which all nodes are greater than or equal to each of its children (heap property). Max-heap can be implemented using a simple k-sized array without allocating any space for pointers: If the tree root is at index 0, then each element at index i ∈ [0, k) has children at indices 2i + 1 and 2i + 2 and its parent is located at index ⌊(i−1)/2⌋. Since the first node contains the largest element, the core pipeline followed by max-array is not altered. Both inserting operations to an empty or a full heap modify the heap to conform to the shape property first, by adding nodes from the end of the heap or replacing the heap root (O(1)). Then, the heap property is restored by traversing up-heap or down-heap (O(k)). Pseudocode for both insertion
functions is shown in Algorithm 2, where \( P(f) \) defines the parent of a fragment \( f \) and \( L(f) \) and \( R(f) \) its left and right children. Figure 3 illustrates how both data structures with \( k = 8 \) are constructed and updated from a number of out-of-order fragment insertions. A representation comparison between max-array and max-heap node pointers is also shown.

![Algorithm 2: InsertToHeap](Image 54x687 to 99x738)

### Algorithm 2 InsertToHeap (Heap \( h \), Pixel \( p \), Fragment \( f \), Int \( k \))

1. **procedure** UP-HEAP\((h, p, f, k)\)
2. \( i := 0; \)
3. \( h[p].\text{counter} := f; \)
4. while \( i < \log_2(k) \) do
5. \( \text{if } f.x > P(f).z \text{ then} \) \( \text{Swap}(f, P(f)); \)
6. \( \text{else} \) break;
7. \( \text{end if} \)
8. \( \text{end while} \)
9. **end procedure**

10. **procedure** DOWN-HEAP\((h, f, k)\)
11. \( i := 0; \)
12. \( h[0] := f; \)
13. while \( i < \log_2(k) \) do
14. \( \text{if } f.x < C(f).z \text{ then} \)
15. \( \text{Swap}(f, C(f)); \)
16. \( \text{end if} \)
17. \( \text{end while} \)
18. **end procedure**

Finally, a sorting process is employed to reorder the fragments for each pixel before generating the final image (resolve pass). Unsorted fragments are initially copied into a local array before performing the depth sort, as it is relatively faster to perform read-write operations in the register space rather in the global graphics memory. Based on the number of captured fragments, a mechanism decides which sorting algorithm is applied to the pixel. Despite its quadratic complexity, insertion sort is faster for sorting small fragment sequences \((k \leq 16)\). When \( k \) increases, \( O(k \log k) \) sorting algorithms, such as shell sort, have better performance [Knowles et al. 2012].

### 3.3 Precise Memory Allocation

Similar to all \( k \)-buffer alternatives where \( k \) is the same for all pixels, \( k \)-buffer suffers from potentially large unused memory space allocation of \( k \)-fragmentless pixels. For example, Figure 2 illustrates the wastefully allocated storage of a 4-buffer for (top) a pixel that consists of 2 fragments and (bottom) an empty-pixel. Note that the value of \( k \) is not automatically adjusted based on the rasterized scene and must be carefully set a priori by the user.

Inspired by S-buffer [Vaslakis and Fudos 2012], we introduce a memory-aware \( k^+ \)-buffer implementation using two geometry passes (\( K^+ \)-B-SB). A precise allocation of the required memory space is achieved by performing an initial geometry rendering (count pass) which sums up the number of fragments covering each pixel. Contrary to S-buffer where all fragments contribute to the per-pixel aggregation, we bound the number of fragments that affect a pixel by \( k \) when \( f(p) > k \), where \( f(p) \) is the number of generated fragments at pixel location \( p[x, y] \). For each incoming fragment, the per-pixel counter is incrementally updated. When the value of the counter reaches \( k \), the subsequently arriving fragments are discarded. The total size of the \( k^+ \)-buffer is estimated by accumulating the bounded per-pixel fragments \( f_s \) using hardware occlusion queries. Then, the memory offset lookup table (referencing pass) is computed in parallel fashion exploiting sparsity in pixel space. To increase memory caching, we cluster non-empty pixels based on an uniform square offset lookup table \((k^+ \)-buffer implementation using two geometry buffers which benefits from the unified framework that successfully integrates the functionality of Z-buffer framework, including the original and supplementary material. Finally, per-pixel counters are reinitialized to zero to guide the subsequent storing phase.

In terms of performance, accessing global memory for concurrently storing all fragments becomes a significant bottleneck as opposed to the original single-pass \( k^+ \)-buffer which benefits from the fast operations in the register memory space. Last but not least, the need of an additional geometry rendering step also adds a tessellation-dependent computation cost. The complete \( k^+ \)-buffer framework, including the original and its memory-aware version, is shown in Figure 4 and Algorithm 3, where \( p.name \) denotes the corresponding member value of pixel structure \( p \). Similarly, \( a[i].name \) defines information located at \( i \) position in the buffer array \( a \), and \( f.name \) denotes attributes of each running fragment \( f \). insert\_empty() and insert\_full() are the abstract insertion functions. max-array and max-heap own versions of both functions are available in the shader source code provided as supplementary material.

### 3.4 Support of Z-buffer and A-buffer

Without loss of generality, \( k^+ \)-buffer can also be considered as a unified framework that successfully integrates the functionality of Z-
buffer, $k$-buffer and A-buffer by simply adjusting the value of $k$. By allocating a single entry per pixel ($k = 1$), our method ensures displaying the closest fragment to the viewer. However, this comes with the additional expense of extra memory requirements and performance downgrade when compared to the hardware depth buffering.

On the other hand, users have to set the value of $k$ large enough to avoid any fragment-overflow ($k = \max_x f(p) \{ f(p) \}$). More specifically, our framework can be considered as a hybrid scheme that correctly simulates the behavior of FreePipe (when $K^+B$ is used) or S-buffer (when $K^+B$-SB is used). Despite the fact that our framework is not restricted from (i) multiple render targets and (ii) samples of the anti-aliasing buffer, a multi-pass variation may be required to achieve a memory-bounded A-buffer functionality. In this case, $\max_x f(p) / k$ rendering iterations have to be performed, resulting at a significant workload increase. Performance-wise, max-array structure should naturally be chosen as the fragment storage due to its constant insertion complexity when the array is not full ($V_f[x, y]: f(p) \leq k$).

Algorithm 3 $k^+\text{-buffer}$ (Array $a$, Pixel $p$, Fragment $f$, Int $k$)

1: procedure CLEAR($p$) $\triangleright$ full-screen pass
2: p.counter := 0;
3: p.semaphore := 0;
4: end procedure

5: procedure COUNT($p$, $k$) $\triangleright$ geometry pass
6: if p.counter < $k$ then
7: p.counter $\leftarrow$ p.counter + 1; $\triangleright$ bounded fragment accumulation
8: else discard;
9: end if
10: end procedure

12: procedure REFERENCING($p$) $\triangleright$ full-screen pass
13: compute_pixel_offset(p.counter);
14: p.counter := 0;
15: end procedure

16: procedure STORE($a$, $p$, $f$, $k$) $\triangleright$ geometry pass
17: if p.counter < $k$ or $f \leq a[0].z$ then $\triangleright$ fragment culling
18: while true do
19: if (p.semaphore $\leftarrow$ 1) $\leftarrow$ 0 then
20: if p.counter < $k$ then $\triangleright$ enter critical section
21: insert_empty(p.counter + ++);
22: else if $f \leq a[0].z$ then $\triangleright$ insert_tail();
23: end if $\triangleright$ exit critical section
24: p.semaphore := 0;
25: end if
26: end while
27: end if
28: end procedure

31: procedure RESOLVE($p$) $\triangleright$ full-screen pass
32: if p.counter $\leq$ 16 then
33: insertion_sort(p.counter);
34: else
35: shell_sort(p.counter);
36: end if
37: compute_effect(p.counter);
38: end procedure

$\triangleright$ where $\{+, -\}$ denote atomic {store, exchange} operations

4 Results

We present an experimental analysis of our $k^+\text{-buffer}$ approach against a set of $k$-buffer and A-buffer realizations focusing on performance, robustness, and memory requirements under different testing conditions. We have measured performance in terms of frames per second (fps) and milliseconds (ms) and memory requirements in terms of megabytes (MB). For the purposes of comparison, we have developed two variations of $K^+AB_{\text{LL}}$, where instead of using per-pixel linked lists for the A-buffer construction, we have applied either fixed-length ($K^+AB_{\text{Array}}$) or variable-length ($K^+AB_{\text{GP}}$) arrays for each pixel. The shader source code from all testing methods has been also provided as supplementary material. All methods are implemented using OpenGL 4.3 API and performed on a NVIDIA GTX 480 graphics card (GeForce 320.49 drivers) with 1.5 GB memory.

Figure 1 demonstrates the importance of accurately handling multi-fragments for several applications (transparency effects [Maule et al. 2011], csg operations [Rossignac et al. 2013], and collision detection [Jang and Han 2008]). Table 1 presents a comparative overview of all $k$-buffer alternatives with respect to memory requirements, rendering complexity, and other features.

4.1 Performance Analysis

We have performed an experimental performance evaluation of our methods against competing techniques using a collection of scenes under several different configurations. Instead of rendering scenes under different image resolutions, we have used a $854 \times 480$ viewport and perform zooming operations defining the percentage of image being rasterized. For a fair comparison, all methods are tested under artificially generated scenes that cover a percentage of screen size (or pixel density: $p_d$) and produce $n = r \cdot k$ randomly arrived fragments per pixel, where $r \geq 1$.

4.1.1 $k$-buffer Comparison

Impact of $k$. Figure 5 shows how the computation time, for each rendering pass of a set of $k$-buffer methods, scales by increasing the value of $k = 4, \ldots, 64$ for a scene that consists of $n = 128$ fragments per-pixel. Except from KB-MDT which needs two passes (namely, Store(Z) and Store), the other memory-bounded methods require to rasterize the scene only once. We observe that our $K^+B$-Array and $K^+B$-Heap perform better than the rest of the techniques for all $k$ values. As expected, $K^+B$-Heap performs better than $K^+B$-Array when moving from low to high $k$ values. Note that the Resolve step is more expensive for A-buffer-aware methods, since it has to locate the closest $k$ fragments from all captured ones before sorting. KB-LL and KB-MDT do not perform depth reordering since they both store and sort fragments on the fly. Despite the good performance of KB-MDT for small buffer sizes, the computation cost of storing and sorting depth fragments exhibits a rapid increase for larger $k$ values. This leads to an important conclusion: a future single-pass KB-MDT, which will be enhanced by the expected 64-bit atomic updates on the graphics memory, will present an insignificant performance gain. Note that $Count$ and $Resolve$ passes of $K^+B$-SB cost less in terms of computations as compared to the ones of $KB-AB_{\text{GP}}$ due to the restricted operations carried out by the former. However, slow fragment storing in global memory from $K^+B$-SB increases fragment spinning which subsequently results at a performance downgrade when the rasterized fragments are significantly increased.

Impact of Sorting. Figure 6 illustrates performance comparison of our bounded $K^+$B against KB and KB-SR methods for varying $k$ values. All methods are tested under two scenarios where $n = k, \ldots, 1024$ fragments are generated for the
$p_a = \{25\%, 75\%\}$ of all pixels. To accurately capture the closest fragments without RMW hazards from KB and KB-SR, the scene is rasterized in depth order. K$^*$ B-Array performs slightly better than K$^*$ B-Heap since the first $k$ fragments need $O(1)$ time (compared to $O(\log_d k)$) to be inserted in the array. Due to the sorted arrival fragment order, the remaining fragments do not affect performance since they are successfully culled in both methods. A linear behavior is observed when moving from less ($0.21M, \ldots, 104.93M$) to more ($0.61M, \ldots, 314.82M$) generated fragments. Our methods are superior when compared with KB for all scene configurations. Similar performance conclusions can be made for the not-implemented KB-PS and KB-MHA methods, since they are supposed to perform slightly worse than KB. Despite its efficiency due to the hardware-implemented stencil routing, KB-SR speed is falling when $k$ reaches higher values. An additional geometry pass must be employed to successfully capture all fragments for KB ($k > 16$) and KB-SR ($k > 32$), resulting in a significant performance downgrade.

**Impact of Memory.** Figure 7 illustrates the performance evaluation in terms of fps per MB for a testing $k$-buffer method set when performance and memory are of utmost importance. To construct $k$-fragmentless pixels, we permit pixels to be influenced by up to $n = 10 - k$ fragments. Thus, we define $f_p$ as the probability of a generated fragment not to be discarded. We observe that K$^*$ B-SB is preferred to be used for handling scenes with many empty pixels ($p_d = 25\%$) and small number of rasterized fragments ($f_p = 25\%$). When pixel and fragment densities increase ($p_d = 75\%, f_p = 75\%$), K$^*$ B-SB performs better than the rest memory-aware methods. However, K$^*$ B-SB behavior is normally worst than the bounded methods since it theoretically performs slower (e.g. one extra pass, storing data at global memory) in conjunction with the small unused memory of the bounded methods. Despite the fast speed of KB-AB$_{SB}$ on sparse scenes, performance is significantly reduced when generated fragments blast off to high levels. Finally, KB-AB$_{Array}$, KB-AB$_{LL}$, KB-AB$_{SB}$ and KB-LL fail to work when fragment allocation results in memory overflow ($k = 64$).

**Figure 7: Performance evaluation in fps/MB (log$_2$ scale) of all $k$-buffer variants when moving from a scene with $r = 10$ from small number towards a large number of generated fragments.**

**Impact of Tessellation.** Figure 8 illustrates how the performance scales by moving from a low (1 level) to a high (64 level) tessellated scene. A representative set of scenes are used to compare a number of $k$-buffer methods that aim at capturing 8 fragments. Three scenes are generated where the same number of fragments are rasterized $n = \{16, 40, 80\}$ for $p_d = 50\%$ of the pixels. Different configurations yield similar speed results. In all tests, a small performance impact is observed from the SB-aware methods as opposed to the linear behavior of the rest $k$-buffers. This is due the fact that the counting geometry pass is not-tessellation dependent.

**Figure 8: Performance evaluation in fps (log$_2$ scale) of all $k$-buffer versions on a scene with varying tessellation resolution $[1, 64]$ and increasing per-pixel fragment complexity $n : r = \{2, 5, 10\}, k = 8$.**

**4.1.2 A-buffer Comparison**

Figure 9 illustrates performance comparison of our methods against the A-buffer alternatives for a scene with varying depth complexity. The same scene configurations with Figure 7’s test have been used, with the difference that $k$ is set to the fragment cardinality, so that K$^*$ B methods are able to capture all generated fragments. We initially observe that both bounded K$^*$ B methods perform better from all memory-aware A-buffer variants and slightly worse than FreePipe, the fastest A-buffer implementation so far. The unnecessary culling mechanism is responsible for this burden. Similar to previous tests, K$^*$ B-Array outperforms K$^*$ B-Heap, enhanced by its constant-time insertion process. On the other hand, K$^*$ B-SB despite its smaller computational cost as compared to LL is worse than SB in all cases. Except from the culling cost, the additional condition for each fragment at the count pass significantly affects performance. Finally, note that the performance gap exhibited between K$^*$ B-SB and KB-AB$_{SB}$ is alleviated when the pixel density is increased (resulting at more rasterized fragments, from $0.21M, \ldots, 13.12M$ to $0.61M, \ldots, 39.35M$).

**Figure 9: Performance evaluation in fps (log$_2$ scale) of A-buffer alternatives on a scene with varying maximum depth complexity.**
We would like to thank AIM@SHAPE Shape Repository for all available methods that more or less simulate the behavior of our framework as compared to previous work. We have introduced \( k^-1 \)-buffer, an improved GPU-accelerated k-buffer framework, which handles RMW memory-hazards and depth precision conversion artifacts, and avoids geometry pre-sorting and the requirement for unbound memory. An additional geometry razerization may be carried out to enable precise memory allocation. Implementation details and light-weight modifications are offered to enable full support of our framework on Fermi and Haswell GPUs. Furthermore, Z-buffer and A-buffer functionalities are successfully integrated under the proposed framework. An extensive experimental comparison has demonstrated the superiority of our framework as compared to previous k-buffer alternatives with regards to storage requirements, performance and image quality.

Further directions may be explored for tackling the problem of multi-fragment rendering in rasterization architectures. Additional experiments showing the performance trade-off between software (spin-lock) and hardware (pixel sync) implemented semaphores on Haswell GPU should further be conducted. Considering completeness, a dynamic k-buffer technique, where \( k \) is not the same for all pixels, should be investigated. In cases, where the number of fragments varies when the camera or scene is animated, an interesting alternative is to capture the \( \% \) of the generated fragments per-pixel. Finally, another challenge is to improve image quality by making use of anti-aliasing algorithms.

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\[
 f(p) = \# \text{fragments at pixel } p(x,y) \\
 f_s(p) = f(p) < k \? f(p) : k \\
 n = \max_i(f(p)) \\
 \frac{f_s(p)}{f(p)} \leq k \\
\]

Table 1: Comprehensive comparison of the prior k-buffer approaches and the introduced k+ buffer methods.


